**Assignment for class 12**

**Mathematics**

**Topic :** *Martins rule or matrix Method for solving the system of Non-Homogeneous linear equation in two variables and three variables having unique solution*

**Application of Determinant and matrices**

- Used for solving the system of linear equations in two or three variables and for checking the consistency of the system of linear equation
- **Consistent system:** A system of linear equation is said to be consistent if it has one or more solution.
- *A consistent system can either have a unique solution or infinitely many solutions.*
- ***A** system of* equation is said to be consistent and independent if it has unique solution.
- A system of equation is said to be consistent and dependent if it has infinitely many solutions.
- **Inconsistent system:** A system of linear equation is said to be inconsistent if its solution does not exist.**(To be discussed in the next assignment)**

**1. Solution for Non -Homogeneous system of Two Linear Equations in Two Variables.**

**Note: Explained in the video link provided.**

**Q1.ii) 4x-3y=3; 3x-5y =7**

A= $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$ , X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B= $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$|A| = -20+9 = -11 \neq 0$

$\Rightarrow A^{-1}$ exist and $A^{-1} = \frac{1}{|A|} adjA = \frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$

**As |A| ≠0, the given system is consistent & has unique solution and $X=A^{-1}B$**

$\Rightarrow$ X= $\frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} 6 \\ 19 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix} \Rightarrow x = -\frac{6}{11}, \ y = -\frac{19}{11}$

**2.Non- homogeneous System of Three Linear Equations in three variables .**

**Method:** Let system of Equations be as below:

$a_1x + b_1y + c_1y = d_1$

$a_2x + b_2y + c_2y = d_2$

$a_3x + b_3y + c_3y = d_3$

Let A = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B= $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$,

A is called *coefficient matrix & X is called variable matrix.*

**If A is non singular matrix , then its inverse exist given system of equation is consistent and independent, i.e, it has unique solution.**

AX=B

$\Rightarrow A^{-1}(AX)=A^{-1}B$ (premultiplying with $A^{-1}$ )

$\Rightarrow (A^{-1}A)X = A^{-1}B$ (By associative property)

$\Rightarrow IX= A^{-1}B$

$\Rightarrow X= A^{-1}B$

**This matrix equation provides unique solution for the given system of equation as inverse of a matrix is unique. This method of solving system of equation is called MATRIX METHOD or MARTINS RULE.**

**Q4.iv) x+2y=5, y+2z=8, 2x+z=5**

A= $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & B= $\begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix}$

Since $|A|= 1(1) -2(-4) =9 \neq O$

If A is non singular matrix , then its inverse exist given system of equation is consistent and independent, i.e, it has unique solution

adj A= $\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -4 \\ -2 & 4 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{1}{|A|}$ adj A= $\dfrac{1}{9}\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -4 \\ -2 & 4 & 1 \end{bmatrix}$

since X= $A^{-1}B$

$\Rightarrow$ X= $\frac{1}{9}\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \\ 5 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ = x=1 , y=2, z=3 Ans

Exercise 4.5 Q5

Solve using matrix method: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9$, $\frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52$, $\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0$

Let $\frac{1}{x}$ =u, $\frac{1}{y}$ =v, & $\frac{1}{z}$ =w,

Therefore equations now can be written as u +v +w=9, 2u +5v +7w= 52, 2u+ v -w =0

A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}$, X= $\begin{bmatrix} u \\ v \\ w \end{bmatrix}$, B= $\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$\Rightarrow$ |A|= 1(-12) -1(-16) +1(-8)=-4 ≠0, $\therefore$ $A^{-1}$ Exist,

$\Rightarrow$ adj A= $\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$

$\Rightarrow \therefore A^{-1} = -\frac{1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$

$\because$ X= $A^{-1}$ B

$\Rightarrow$X= $-\frac{1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$,

$\Rightarrow$ u= $\frac{1}{x}$ =1 $\Rightarrow$ x=1, v= $\frac{1}{y}$ =3 $\Rightarrow$ y= $\frac{1}{3}$ , w= $\frac{1}{z}$ =5 $\Rightarrow$ z= $\frac{1}{5}$

**Homework: Q1. i) Q4. ii) ,iii), v)Q6, Q7. Q8. Q9**

**NOTE: Q1ii)Q2i) Q4.i)10, Q.11, & Q12. has been discussed in the video link provided to you by the school**